Quantitative Literacy:
Thinking Between the Lines

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Chapter 2:
Analysis of Growth

Chapter 2: Analysis of Growth
Lesson Plan

- Measurements of growth: How fast is it changing?
- Graphs: Picturing growth
- Misleading graphs: Should I believe my eyes?

Learning Objectives:

- Understand intuitive notion of functions
- Read data table and calculate the percentage change
- Calculate the average growth rate
- Estimate by interpolation and extrapolation from a function value

2.1 Measurements of growth: How fast is it changing?

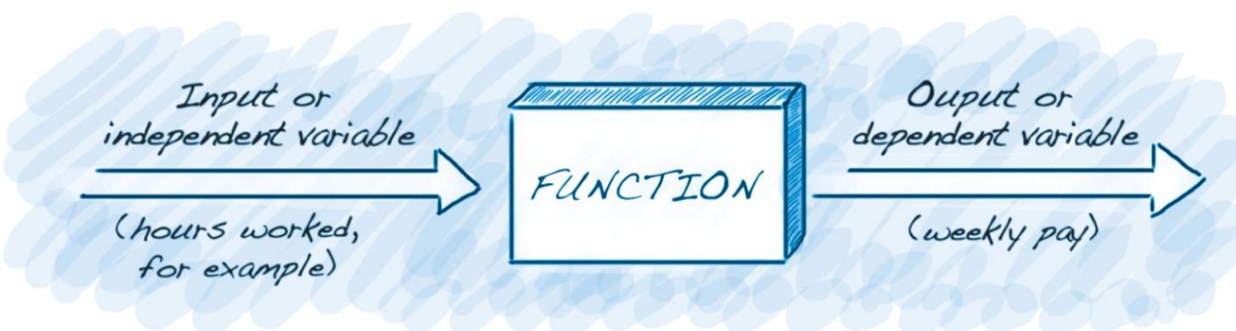
- When one quantity (or variable) depends on another, the latter is referred to as the **independent variable** and the former is referred to as the **dependent variable**.
- A **function** describes *how* the dependent variable *depends on* the independent variable.

2.1 Measurements of growth: How fast is it changing?

- The **Independent variable =** the **input** value
- The **dependent variable =** the **output** value of a function.

- **Example:** If you work for an hourly wage, your pay for the work depends on the number of hours you work.

FIGURE 2.2 Visual representation of a function.



- **Example:** The following table shows the most medals won by any country in a given year of the Olympic Winter Games:

Year	Most medals won by a country	Country
1988	29	Soviet Union
1992	26	Germany
1994	26	Norway
1998	29	Germany
2002	36	Germany
2006	29	Germany
2010	37	United States

What would you label as the independent and the dependent variables for this table?

- **Solution:**

The year = the Independent variable

The number of medals won = the dependent variable

- **Note:** the date cannot be a dependent variable because there were 3 different years in which 29 medals were won. The year does **not** depend only on the number of medals won.

- Thus, the year is **not a function** of the number of medals won.

- The **percentage change** (or **relative** change) in a function is the percentage increase in the function from one value of the independent variable to another.

$$\text{Percentage change} = \frac{\text{Change in function}}{\text{Previous function value}} \times 100\%$$

- **Example:** Calculate the percentage change in the U.S. population from 1790 to 1800. Round your answer in the nearest whole number.

2.1 Measurements of growth: How fast is it changing?

- **Solution:**

TABLE 2.1 Population of the United States

Year	Population (millions)	Year	Population (millions)
1790	3.93	1900	76.21
1800	5.31	1910	92.23
1810	7.24	1920	106.02
1820	9.64	1930	123.20
1830	12.87	1940	132.16
1840	17.07	1950	151.33
1850	23.19	1960	179.32
1860	31.44	1970	203.30
1870	38.56	1980	226.54
1880	50.19	1990	248.71
1890	62.98	2000	281.42
		2010	308.75

$$\text{Percentage change} = \frac{\text{Change in function}}{\text{Previous function value}} \times 100\%$$

$$= \frac{\text{Change from 1790 to 1800}}{\text{Population in 1790}} \times 100\%$$

$$= \frac{5.31\text{million } - 3.93 \text{ million}}{3.93 \text{ million}} \times 100\% = 35\%$$

2.1 Measurements of growth: How fast is it changing?

- **Example:** The following table shows the world population (in billions) on the given data:

Date	1950	1960	1970	1980	1990	2000
Population (billions)	2.56	3.04	3.71	4.45	5.26	6.08

1. Identify the independent variable and the function, and make a bar graph that displays the data.

2. Make a table and bar showing percentage changes between decades.

- **Solution:** 1. The independent variable = the date.

  The function = the world population on that date.

2. The percentage increase from 1950 to 1960 is

$$\text{Percentage change} = \frac{\text{Change in function}}{\text{Previous function value}} \times 100\%$$
$$= \frac{3.04 - 2.56}{2.56} \times 100\% = 19\%$$

2.1 Measurements of growth: How fast is it changing?

- **Solution:** The table is given below, and the bar graph is in Figure 2.6.

Decade	1950-1960	1960-1970	1970-1980	1980-1990	1990-2000
% change	19%	22%	20%	18%	16%

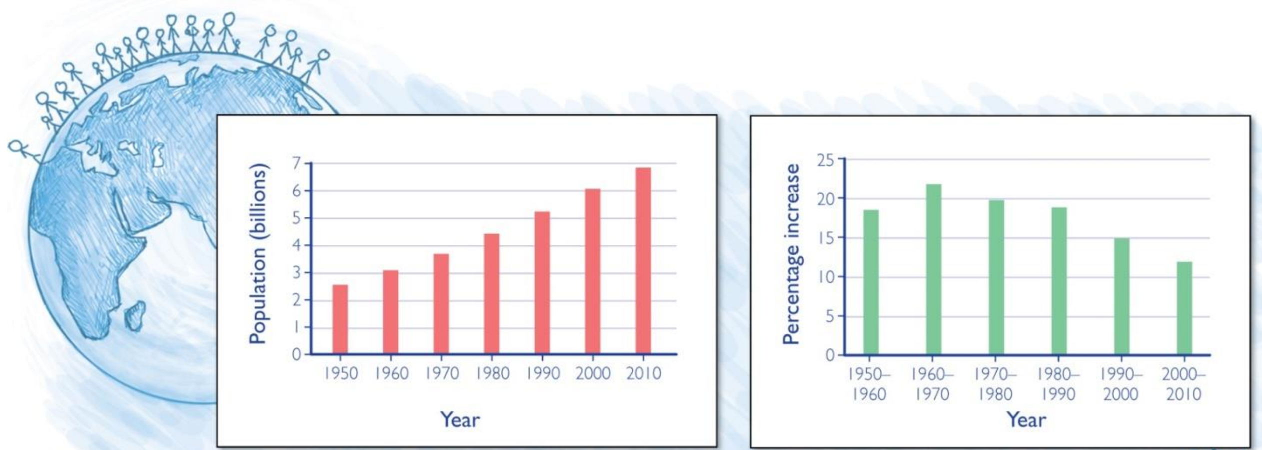


FIGURE 2.5 World population.

FIGURE 2.6 Percentage growth of world population.

2.1 Measurements of growth: How fast is it changing?

- The **average growth rate** of a function over an interval is the change in the function divided by the change in the independent variable.

$$\text{Average growth rate} = \frac{\text{Change in function}}{\text{Change in independent variable}}$$

- **Example:** The population of Russia declined from about 146 million in 2000 to about 143 million in 2007. Calculate the average growth rate over this period and explain its meaning.

- **Solution:** The change in population is negative,

143 – 146 = –3 million. The change in time is 7 years.

$$\text{Average growth rate } = \frac{\text{Change in function}}{\text{Change in independent variable}}$$
$$= -\frac{3}{7} = -0.429 \text{ million per year}$$

- This means that over this interval the population declined, on average, by about 429,000 per year.

- **Example:** Assume that the independent variable is the year and the function gives the tuition cost, in dollars, at your university.

  Give the units of the average growth rate and explain in practical terms what that rate means.

- **Solution:** The change in the independent variable is the elapsed time measured in years.

- The change in the function is the tuition increase measured in dollars.

- The units of the average growth rate are dollars per year.

- The average growth rate means how much we expect the tuition to increase each year.

- **Interpolation** is the process of estimating unknown values between known data points using the average growth rate.

- **Example:** In the fall of 2005, 37.7% of college freshmen in the United States believed that marijuana should be legalized.

In the fall of 2008, that figure was 41.3%. Use these figures to estimate the percentage in the fall of 2007.

The actual figure for 2007 was 38.2%.

What does this say about how the growth rate in the percentage varied over time?

- **Solution:** The change in the independent variable from 2005 to 2008 is three years.

- The change in the dependent variable over that period is

$41.3 - 37.3 = 3.6$ percentage points.

$$\frac{\text{Change in function}}{\text{Change in independent variable}} = \frac{3.6}{3} = 1.2$$

- Hence, the average growth rate from the fall of 2005 to the fall of 2008 was 1.2 percentage points per year.

- **Solution:**

- The percentage in 2007 was about:

$$\text{Percentage in } 2005 + 2 \text{ years of increase} = 37.7 + 2 \times 1.2 = 40.1\%$$

- Our estimate of 40.1% for the fall of 2007 is higher than the actual figure of 38.2%. The figure grew more quickly from 2007 to 2008.

- **Example:** Severe Acute Respiratory Syndrome (SARS) is a viral respiratory disease. There was a serious outbreak initially in China from November 2002 to July 2003. There were 8096 known cases and 774 deaths worldwide.

1. Calculate the average growth rate of cases from March 26 to March 31.

2. Use your answer from part 1 to estimate the cumulative number of SARS cases by March 28. The actual cumulative number on March 28 was 1485. What does this say about how the growth rate varied over time?

- **Solution:** The table from the World Health Organization shows the cumulative number of SARS cases:

Date	March 26	March 31	April 5	April 10	April 15
Number of cases	1323	1622	2416	2781	3235

1. The independent variable is the data.

The function is the cumulative number of SARS cases reported.

$$\frac{\text{Change in reported cases}}{\text{Elapsed time}} = \frac{1622 - 1323}{5} = 59.8$$

The average growth rate from March 26 to March 31 is 59.8 new cases per day.

- **Solution:** 2. There were 1323 cases on March 26, and we expect to see about 59.8 new cases on March 27 and again on March 28.

$$\text{Estimated cases on March 28}$$
$$= \text{Cases on March 26} + 2 \times \text{Average new cases per day}$$
$$= 1323 + 2 \times 59.8$$
$$= 1442.6.$$

Thus, estimated cases on March 28 is about 1443 cases.

Our estimate using interpolation is relatively good but somewhat lower than the actual value of 1485.

- **Extrapolation** is the process of estimating unknown values beyond known data points using the average growth rate.

- **Example:** The following table shows the average age, in years, of first-time mothers in the given year.

Year	1970	1980	1990	2000
Average age	21.4	22.7	24.2	24.9

1. Estimate the average age of first-time mothers in 1997.
2. Predict the average age of first-time mothers in 2005.
3. Predict the average age of first-time mothers in the year 3000. Explain why the resulting figure is not to be trusted.

- **Solution:**

1. We estimate the average age in 1997 **by interpolating**. The average growth rate between 1990 and 2000 is:

$$\text{Average growth rate} = \frac{24.9 - 24.2}{10} = 0.07$$

- So, the average age of first-time mothers increased at a rate of 0.07 year per year over this decade.
- The average age in 1997 is estimated to be

  $24.7 + 7 \times 0.07 = 24.69$ years, or about 24.7 years.

- **Solution:**

2. We estimate the average age **by extrapolating**.

- The average growth rate between 1990 and 2000, that we found in part 1 to be 0.07 year per year.

- Thus, we estimate an increase of 0.07 year over each of the five years from 2000 to 2005:

$\text{Estimate average age in } 2005 = 24.9 + 5 \times 0.07 = 25.25$ years

Or about 25.3 years.

- **Solution:**

3. This is exactly like part 2.

- The average growth rate from 1990 to 2000 is 0.07 year per year.
- The year 3000 is 1000 years from the year 2000. This growth rate gives a prediction for the year 3000 of:

$$24.9 + 1000 \times 0.07 = 94.9 \text{ years}$$

- Our projection for the average age in the year 3000 of first-time mothers is 95 years.
- The number is silly and clearly illustrates the danger of extrapolating too far beyond the limits of the given data.

Learning Objectives:

- Study various types of graphs: bar graphs, scatterplots, and line graphs
- Learn how to interpret the patterns
- Analyze the graphs critically
- Determine advantages and disadvantages of each type of graph

2.2 Graphs: Picturing growth

- **Graphs** allow us to visualize information, which helps see patterns that may not be readily apparent from a table.
- **Example:** A graph reveals visually where the function is increasing or decreasing and how rapid the change is.
- A **scatterplot** is a graph consisting of isolated points, with each dot corresponding to a data point.
- To make a **line graph,** we begin with a scatterplot and join the adjacent points with straight line segments.

- **Example:** The running speed of ants varies with the ambient temperature. Here are data collected at various temperatures:

Temperature (degrees Celsius)	Speed (centimeters per second)
25.6	2.62
27.5	3.03
30.4	3.56
33.0	4.17

First make a scatterplot of the data showing the speed as the function and the temperature as the independent variable, then make a line graph using these data.

2.2 Graphs: Picturing growth

- Solution: To make a scatterplot, we plot the data points on the graph: (25.6.2.62),…, (33.0, 4.17).

We join the points with line segments to get the line graph.

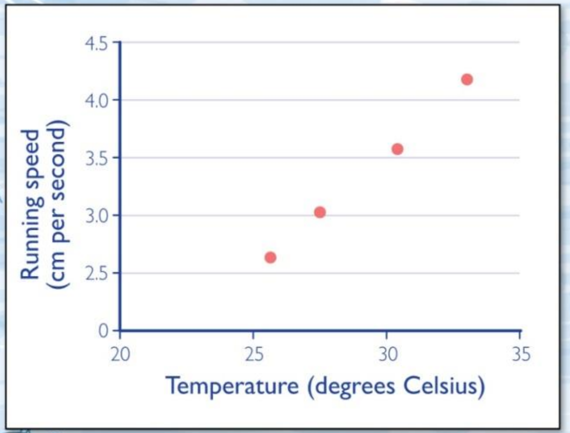


FIGURE 2.15 Running speed of ants versus temperature: scatterplot.

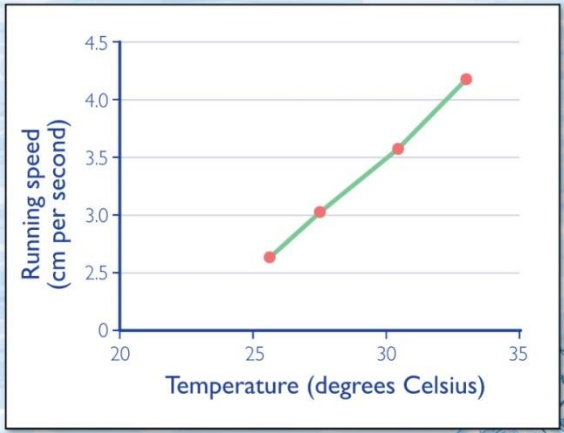


FIGURE 2.16 Running speed of ants versus temperature: line graph.

- **Interpreting line graphs: Growth rates and graphs**

1. The growth rate of data is reflected in the steepness of the graph. Steeper graphs indicate a growth rate of greater magnitude.

2. An increasing graph indicates a positive growth rate, and a decreasing graph indicates a negative growth rate.

- **Example:** The line graph in Figure 2.18 shows the yearly gross income in thousands of dollars for a small business from 2002 through 2011. Explain what this graph says about the rate of growth of yearly income.

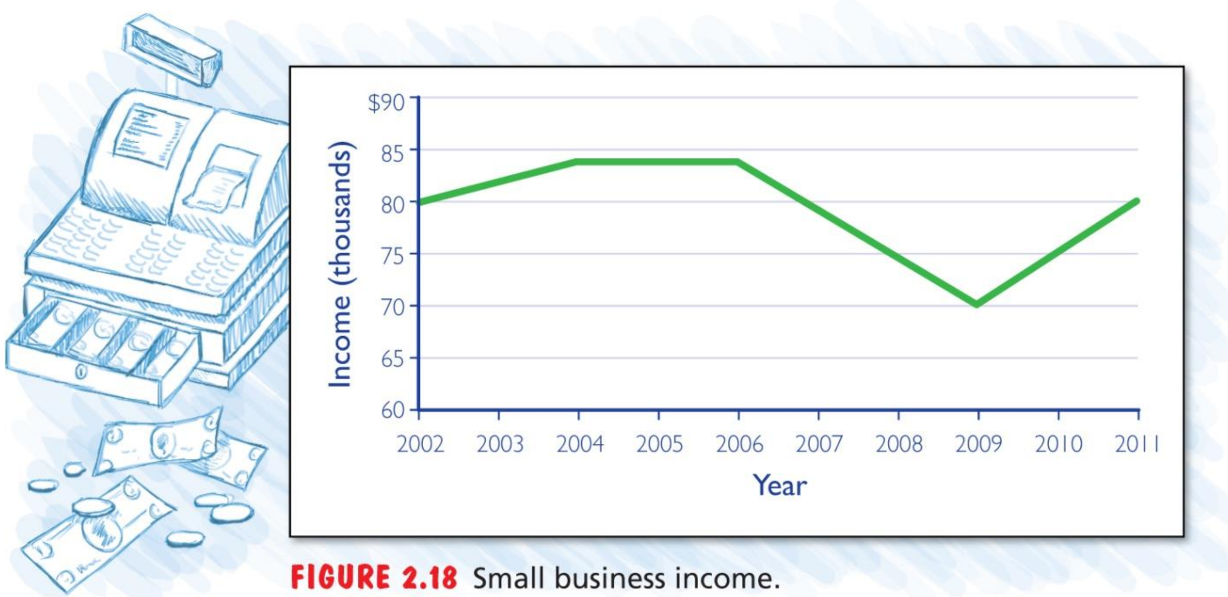


FIGURE 2.18 Small business income.

- A **smooth line graph** is made from a scatterplot by joining data points smoothly with curves instead of line segments.
- **Example:** The graph of the amount of toxic waste remaining as a function of time is decreasing at a decreasing rate. Sketch an appropriate graph for the amount of toxic waste remaining as a function of time.
- **Solution:**

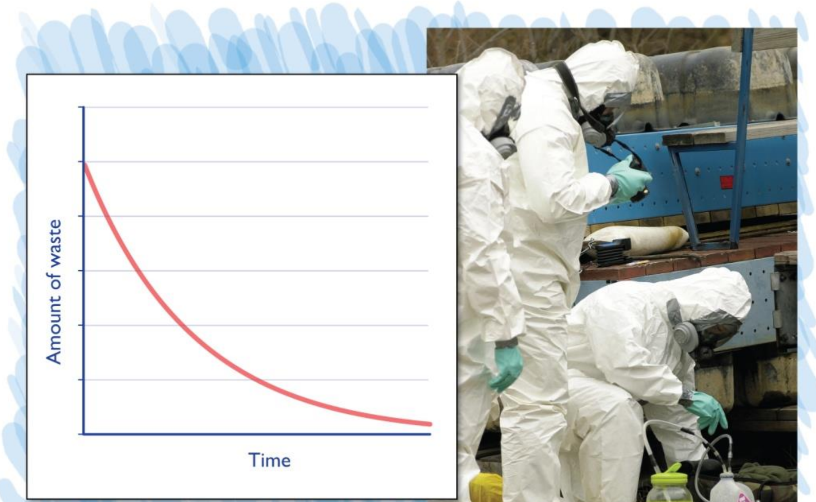


FIGURE 2.23 Toxic waste disposal from a meth lab site.

2.2 Graphs: Picturing growth

- In **practical settings,** the **growth rate** has a familiar meaning.
- **Example:** The graph in Figure 2.24 shows a population that increases from a time and then begins to decrease. The growth rate in this context is the rate of population growth. The growth rate is positive in Year0. It remains positive until we reach Year3, where the growth rate is 0. Beyond Year3 the growth rate gets negative through Year5.

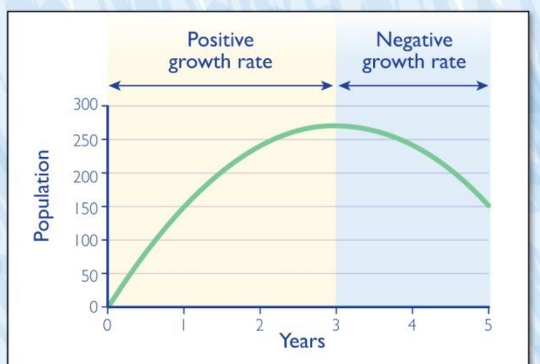


FIGURE 2.24 A population graph.

Learning Objectives:

- Understand common types of misleading graphs:

1. Misleading by choice of axis scale
2. Default rages on the graphs generated by calculators and computers
3. Misleading by misrepresentation of data: Inflation
4. Misleading by using insufficient data
5. Pictorial representations

- **Example (Analyzing a choice of scale):** An article from the Media Matters Web site asserts: In presenting the results of CNN/USA Today/Gallup poll, CNN.com used a visually distorted graph that falsely conveyed the impression that Democrats far outnumber Republicans and Independents in thinking a Florida state court was right to order Terri Schiavo's feeding tube removed.

- CNN presented the graph shown in Figure 2.37 to show the response to the following question:

"Based on what you have read about the case, do you agree with the court's decision to have feeding tube removed?"

- **Example (Analyzing a choice of scale):** CNN.com responded to criticism by replacing Figure 2.37 with Figure 2.38 at its Web site.

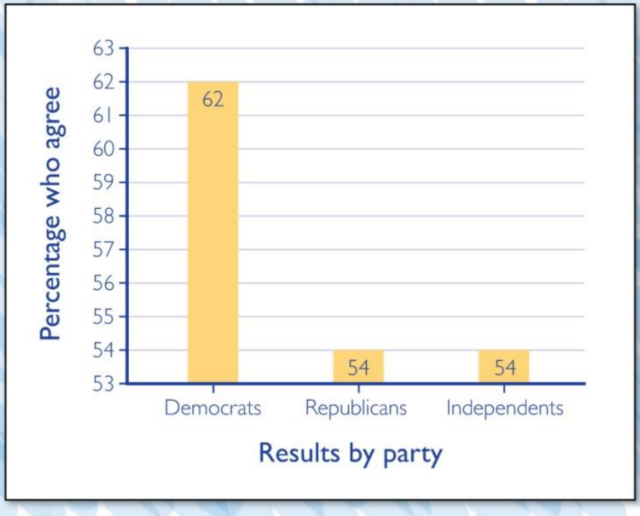


FIGURE 2.37 Graphic initially posted by CNN.com.

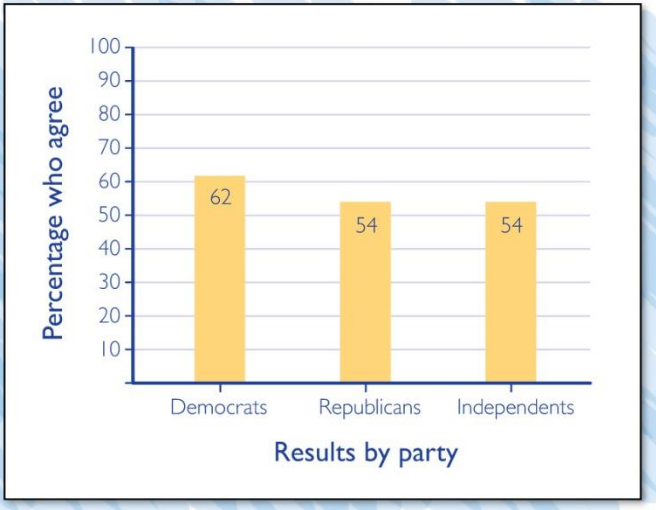


FIGURE 2.38 Replacement graphic posted by CNN.com.

- **Example (Analyzing a choice of scale):**

1. What percent of Republicans polled agreed with the courts decision? What percent of Democrats polled agreed with the court's decision?

2. Fill in the blank: *The fraction of Democrats agreeing with the court's decision was _____ percent more than the fraction of Republicans agreeing with the court's decision.*

3. What impression does CNN's original graph give about how Democrats and Republicans respond to this question 2?

- **Solution:** 1. The graph shows that 54% of Republicans polled agreed with the court's decision and 62% of Democrats polled agreed with the court's decision.

2. Percentage change

$$= \frac{\text{Difference in percentages}}{\text{Republican percentage}} \times 100\%$$

$$= \frac{8}{54} \times 100\% = 15\%$$

The fraction of Democrats agreeing was 15% percent more than the fraction of Republicans agreeing with the court's decision.

- **Solution:**

3. Figure 2.37: the bar representing Democrats is 9 units high; the bar representing Republicans is 1 unit high.

The fraction of Democrats agreeing was 9 times as large as the fraction of Republicans agreeing; that is 800% more instead of 15% more.

- **Calculation tip (Graphing on Calculators):**

A graphing calculator (or computer software) has **default settings** for the scales it uses on the two axes.

When we plot a graph with a calculator (or computer) we need to be alert to those scales.

We might need to **adjust** them manually to obtain a graph that conveys an appropriate picture.

- **Example (Choosing scales for graphs):**

The table shows the average price per gallon of regular gasoline in January of the given year:

Year	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Average Price	$1.28	$1.10	$0.93	$1.27	$1.40	$1.09	$1.44	$1.51	$1.77	$2.24	$2.33

Your proposition: "The average price per gallon of regular gasoline showed a significant increase from 1997 to 2007."

Make a bar graph of the data that you would use for an argument in support of the proposition. Make a second bar graph that you would use for an argument against the proposition.

2.3 Misleading graphs: Should I believe my eyes?

- **Solution:**

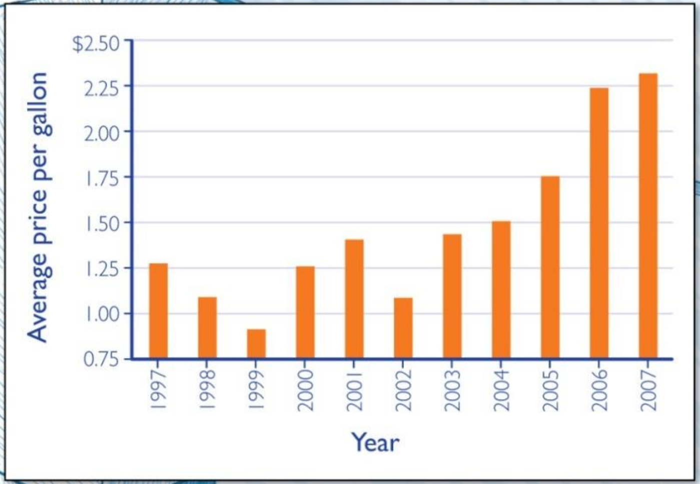


FIGURE 2.40 Emphasizing the increase in gasoline prices.

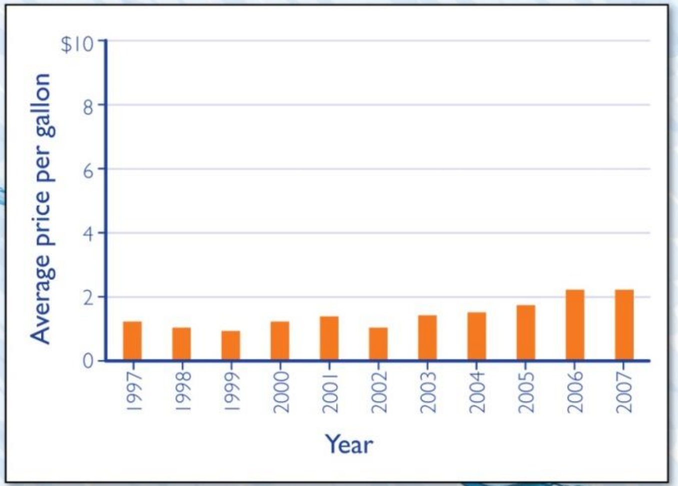


FIGURE 2.41 Deemphasizing the increase in gasoline prices.

Adjusting for Inflation

- It is important to know whether graphs involving currency are adjusted for inflation.
- If inflation from Year1 to Year2 is r as a decimal,

$$\text{Year1 one dollar} = \text{Year2 } (1 + r) \text{ dollars}$$

- If inflation from Year1 to Year2 is r as a decimal, express Year1 dollars in constant Year2 dollars using:

Constant-dollars formula

$$D \text{ "Year}-1\text{ dollars"} = D(1 + r)\text{"Year} - 2\text{ dollars"}$$

- **Example (Adjusting for inflation):** The following table shows the average cost per gallon of regular gasoline in the given year. These data are plotted in Figure 2.45.

Year	1960	1970	1980	1990	2000
Price per gallon	$0.31	$0.36	$1.25	$1.16	$1.51

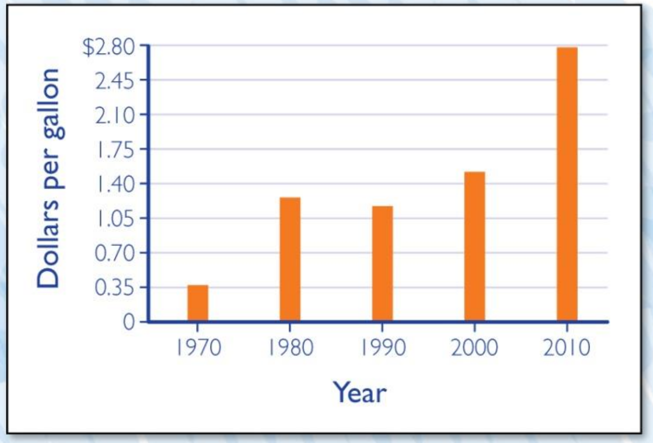


FIGURE 2.45 Gasoline prices.

- **Example (Adjusting for inflation):** Inflation rates are shown in the following table:

Time span	1960-2000	1970-2000	1980-2000	1990-2000
Inflation	484%	337%	102%	30%

1. Use the graph in Figure 2.45 to determine in what year gasoline was most expensive.
2. Complete the following table showing gasoline prices in constant 2000 dollars:

Year	1960	1970	1980	1990	2000
Price per gallon	$0.31	$0.36	$1.25	$1.16	$1.51
Price in 2000 dollars					

3. Make a bar graph showing gasoline price in constant 2000 dollars.

- **Solution:**

1. Gasoline was most expensive in 2000.

2. The price per gasoline of 1960 gas in terms of 2000 dollars:

The inflation rate is 484%: $r = 4.84$. We use the constant-dollars formula:

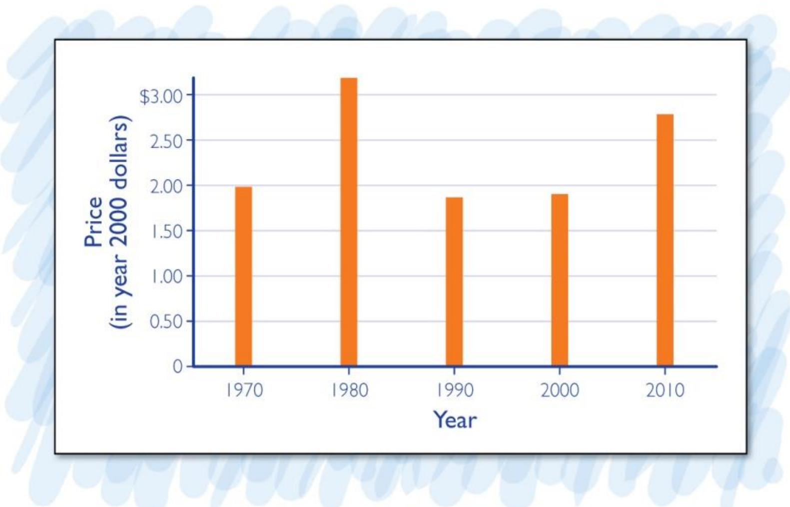
$$\begin{aligned} D \text{ "1960 dollars"} &= D(1+r) \text{ "2000 dollars"} \\ 0.31 \text{ "1960 dollars"} &= 0.31(1+4.84) \text{ "2000 dollars"} \\ &= 1.81 \text{ "2000 dollars"} \end{aligned}$$

So, the price of gas in 1960 was $1.81 in "2000 dollars."

- **Solution:**

Year	1960	1970	1980	1990	2000
Price per gallon	\$0.31	\$0.36	\$1.25	\$1.16	\$1.51
Price in 2000 dollars	0.31 × 5.84 =\$1.81	0.36 × 4.37 =\$1.57	1.25 × 2.02 =\$2.53	1.16 × 1.30 =\$1.51	\$1.51

3.



- **Example (making a pie chart):** make a pie chart showing the following enrollment data from the University of California, Davis:

UC Davis Undergraduates	23,499
Agricultural and Environmental Sciences	4819
Engineering	2950
Letters and Science	10,243
Biological Sciences	5361
Teaching Credential	126

- **Solution:** Percentage of the whole represented by each category of student, that is, 4819 students in Engineering represent

$$\frac{2950}{23{,}499} \times 100\% = 12.6\%$$

UC Davis Undergraduates	23,499	100%
Agricultural and Environmental Sciences	4819	20.5%
Engineering	2950	12.6%
Letters and Science	10,243	43.6%
Biological Sciences	5361	22.8%
Teaching Credential	126	0.5%

- **Solution:**

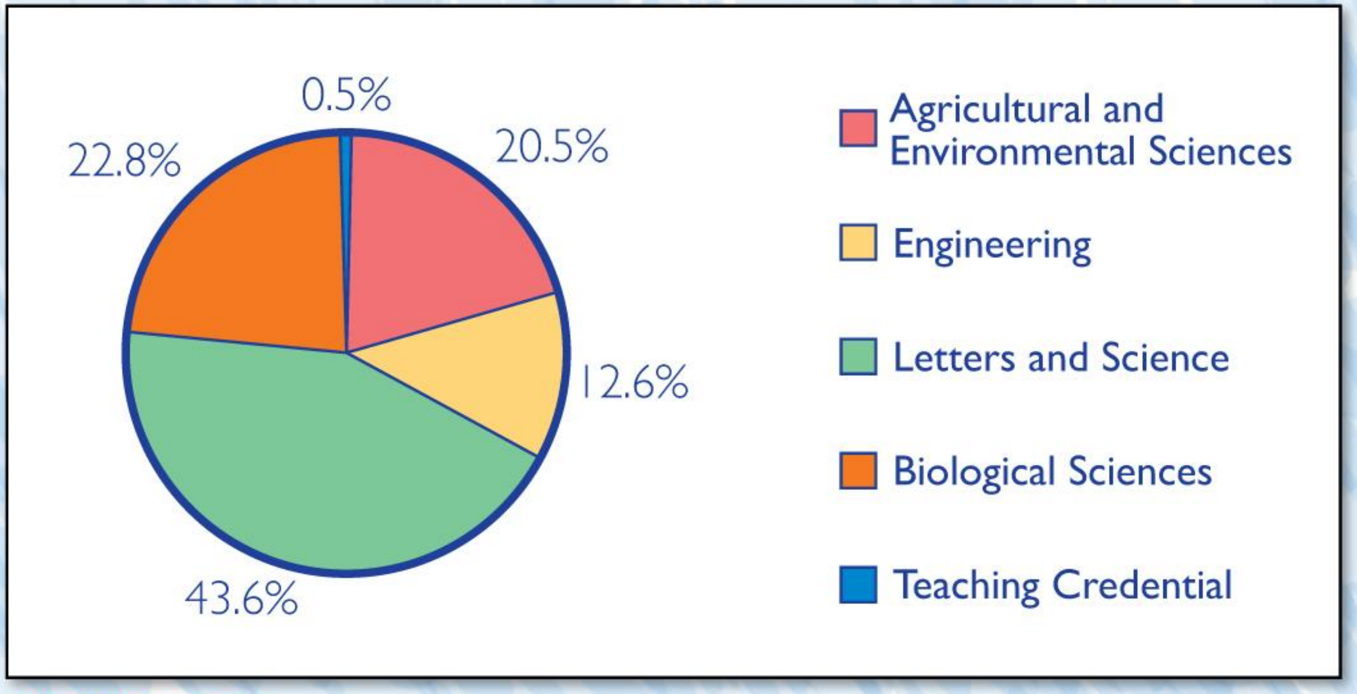


FIGURE 2.50 UC Davis undergraduate enrollment.

Chapter 2 Analysis of Growth: **Chapter Summary**

- **Measurements of graphs**: How fast is it changing?
  - Use growth rates to analyze quantitative information
  - Tables and percentage change:

    A data table, a bar graph

    the Percentage change formula
  - Interpolation and Extrapolation:

    the Average growth formula

    Estimate by interpolation or extrapolation

    from a function value

- **Graphs**: Picturing growth
  - Understand various types of graphs: bar graphs, scatterplots, line graphs, and smoothed line graphs.
  - Growth rates and graphs:

    The growth rate of data is reflected in the steepness of the graph.

    An increasing graph indicates a positive growth rate.

    A decreasing graph indicates a negative growth rate.

Chapter 2 Analysis of Growth: **Chapter Summary**

- **Misleading graphs**: Should I believe my eyes?
  - By choice of axis scale
  - Default ranges on graphs generated by calculators and computers
  - By misrepresentation of data: Inflation

    Adjusting for inflation
  - By using insufficient data
  - Pictorial representations: Pie chart